Mental and Written Calculation Policy

Based on the Early Years Foundation Stage and National Curriculum at Key Stages 1 & 2

Aims:

We aim to ensure that all pupils:

- Become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and develop an argument, justification of proof using mathematical language.
- Can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and preserving in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. Pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems.

The expectation is that the majority of pupils will progress at broadly the same pace, although decisions about when to progress to the next stage should always be based on the security of pupils' understanding. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding before moving on.

Early Years Foundation Stage

The principle focus of mathematics in the Early Years Foundation Stage is to ensure that pupils develop a good understanding of the number system by counting and ordering numbers and using this knowledge to support their understanding of addition and subtraction. This should involve solving simple problems set in practical contexts.

By the end of the Foundation Stage pupils should understand the concepts of addition and subtraction and solve simple calculations of single-digit numbers using objects and by counting on or back.

Key Stage 1

The principle focus of mathematics in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value.

Lower Key Stage 2

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.
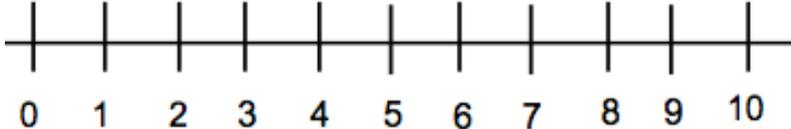At this stage, pupils should develop their ability to solve problems, including with decimal place value. By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Upper Key Stage 2

The principle focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

Problems will demand efficient written and mental methods of calculation. Pupils are introduced to the language of algebra. By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division.

PROGRESSION OF NUMBERLINES

EYFS	**Number track**	Has the numbers inside the sections, rather than on the divisions	
Moving towards end of year 1 expectation	**Calibrated, numbered numberline**	Equal divisions marked on the numberline and each division is numbered	
Year 1 – National age related expectation	**Calibrated, unnumbered numberline**	Equal divisions are marked, but left unnumbered for children to add relevant numbers to	
Year 2 – National age related expectation	**Blank numberline**	No divisions or numbers marked for the children	

Early Years Foundation Stage

For children working at these developmental stages, calculation is not taught in isolation from the other elements of Mathematics or the EYFS curriculum.

30 – 50 Months	40-60+ Months
- Compare two groups of objects, saying when they have the same number. When playing a game, all get 4 counters. Do you have the same as your friend? Let's both get 2 balls and then we can go play. Have you got the same as me? - Show an interest in number problems. Number rhymes with resources, how many frogs in the pond, how many altogether? Give everyone two biscuits from this box, how many are there? Share the biscuits out so that everyone has the same number, how many does everyone get? - Separate a group of three or four objects in different ways, beginning to recognise that the total is still the same. Compare bears- sorting by colour, size- emphasis on same total. Show me five fingers. Use both hands. Show me another way to do it.	- Find the total number of items in two groups by counting all of them. In the field there are four cows and three horses. How many animals are there altogether? - Say the number that is one more than a given number. Show me three fingers, what is one more? Show me six fingers, what is one more? - Find one more or one less from a group of up to five objects, then ten objects. I am going to add one more cube to this set of these four cubes. How many cubes will there be then? I am going to take away one of these five cubes. How many cubes will be left? Take away one flower from this set of 7 flowers. How many flowers are there now? - In practical activities and discussion, begin to use the vocabulary involved in adding and subtracting. Can you tell me what you have been doing? What happens when we put more counters in the pot? - Records, using marks that they can interpret and explain. How could we write down what we are doing? Can you tell me what you have written down?

	- Begins to identify own mathematical problems based on own interests and fascinations.
	What happens if we take some of those toys away?
	How can we solve the problem?

By the end of EYFS children should be able to achieve the Early Learning Goals for calculating in Number of Mathematics

Early Learning Goal

- Say which number is one more or one less than a given number to twenty.

- Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

- Solve problems, including doubling, halving and sharing.

When children are working within this level they are beginning to work at the National Curriculum.

Written methods for addition of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Addition		
Year 1 – Age Related Expectations	**Year 2 – Age Related Expectations**	**Year 3 – Age Related Expectations**

- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds
- add and one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems as seem below
 	
+ = signs and missing numbers
 	
3 + 4 = □   □ = 3 + 4
3 + □ = 7   7 = □ + 4
□ + 4 = 7   7 = 3 + □
□ + ▽ = 7   7 = □ + ▽
 	
Promoting covering up of operations and numbers.
 	
Number lines (blank)
 	
Using blank number lines
 	
(Teacher model number lines with missing numbers)
 	
7 + 4 = 11
	- solve problems with addition:
	- using concrete objects and pictorial representations,
	- applying their increasing knowledge of mental and written methods
	- recall and use addition facts to 20 fluently, and derive and use related facts up to 100
	- add and numbers using concrete objects, pictorial representations, and mentally, including:
	- a two-digit number and ones
	- a two-digit number and tens
	- two two-digit numbers
	- adding three one-digit numbers
	- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
	- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
	
	Partition into tens and ones and recombine
	
	12 + 23 = 10 + 2 + 20 + 3
	        = 30 + 5
	        = 35
	
	refine to partitioning the second number only:
	
	23 + 12 = 23 + 10 +1 + 1
	        = 33 + 1 + 1
	        = 35
		- add and subtract numbers mentally, including:
		- a three-digit number and ones
		- a three-digit number and tens
		- a three-digit number and hundreds
		- add and subtract numbers with up to three digits, using formal written methods of columnar addition
		- estimate the answer to a calculation and use inverse operations to check answers
		- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
		
		Partition into hundreds, tens and ones and recombine
		Either partition both numbers and recombine or partition the second number only e.g.
		358 + 73 = 358 + 70 + 3
		           = 428 + 3
		           = 431
		
		    +70 +3
		
		358 428 431
		
		Pencil and paper procedures:
		83 + 42 = 125
		
		      8 3
		    + <u>4 2</u>
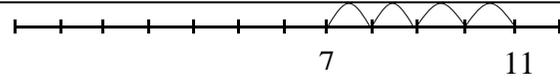
Children go up in 1s

7   11

+10   +1   +1

23   33   35
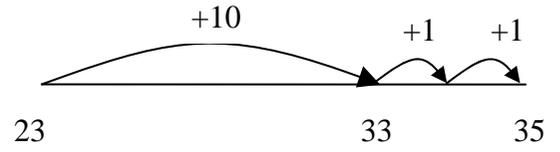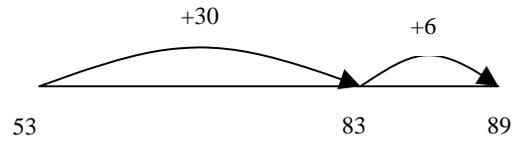
Partition into tens and ones and recombine
Partition both numbers and recombine. Refine to partitioning the second number only e.g.
36 + 53 = 53 + 30 + 6
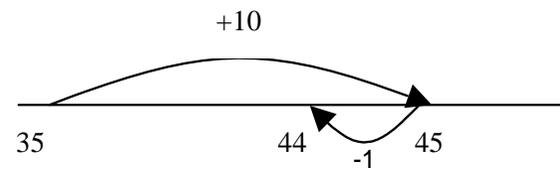   = 83 + 6
        = 89

+30   +6

53   83   89

Add a near multiple of 10 to a two-digit number

Mental Method

Add 9 or 11 by adding 10 and adjusting by 1
35 + 9 = 44

+10

35   44   45
                 -1

5
120
125

  358
+ 73
   11
  120
  300
  431

  £3.42
+ £1.36
  £4.78

Leading to formal method, showing numbers exchanged underneath
  358
+ 73
  431
  1 1

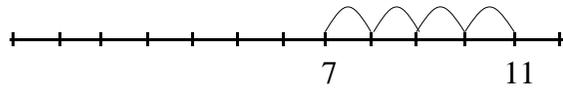| Addition		
Year 4 – Age Related Expectations	**Year 5 – Age Related Expectations**	**Year 6 – Age Related Expectations**
- add numbers with up to 4 digits using the formal written methods of columnar addition and where appropriate - estimate and use inverse operations to check answers to a calculation - solve addition and solve two-step problems in contexts, deciding which operations and methods to use and why.	- add whole numbers with more than 4 digits, including using formal written methods (columnar addition) - add numbers mentally with increasingly large numbers - use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy - solve multi-step problems in contexts, deciding which operations and methods to use and why.	- solve multi-step problems in contexts, deciding which operations and methods to use and why - use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. Algebra - use simple formulae - express missing number problems algebraically - find pairs of numbers that satisfy an equation with two unknowns .
3587 + 675 4262 1 1 1 Extend to decimals (same number of decimals places) and adding several numbers (with different numbers of digits). *Model negative numbers using a number line.*	**Pencil and paper procedures** Extend to numbers with any number of digits and decimals with 1 and 2 decimal places. 124.9 + 117.25 = 242.15 124.90 *put in a zero to secure the place value* + 117.25 242.15 1 1 673.9 + 56.045 729.945 1	

Written methods for subtraction of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Subtraction		
Year 1 – Age Related Expectations	**Year 2 – Age Related Expectations**	**Year 3 – Age Related Expectations**

Year 1 – Age Related Expectations

- read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = – 9.

Pictures / marks

Sam spent 4p. What was his change from 10p?
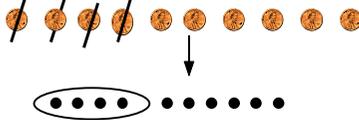
– = signs and missing numbers

7 - 3 = ☐	☐ = 7 - 3
7 - ☐ = 4	4 = ☐ - 3
☐ - 3 = 4	4 = 7 - ☐
☐ - ▽ = 4	4 = ☐ - ▽

Visual / practical activities

Number lines

The difference between 7 and 11

Year 2 – Age Related Expectations

- solve problems with subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- show that subtraction has to be done in the order stated
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Pencil and paper procedures
Complementary addition
84 – 56 = 28

Year 3 – Age Related Expectations

- subtract numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
- subtract numbers with up to three digits, using formal written methods of columnar subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex subtraction.

Use known number facts and place value to subtract

Develop year 2 skills using 3 digit number – 2 digit number

Estimate first….
197 – 15 = 182

182   187   197

-5   - 10
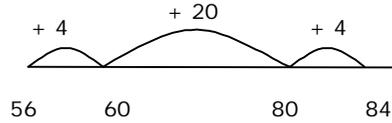
(Counting on)
To reinforce concept. Practical strategies essential to see 'difference'.

7   11

Recording by - drawing jumps on prepared lines
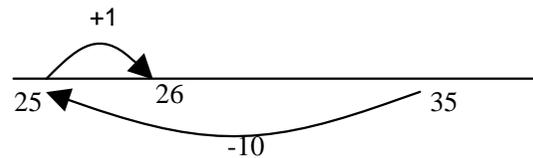   - constructing own lines, if appropriate

(Teachers model jottings appropriate for larger numbers)

+ 4   + 20   + 4

56   60   80   84

Mental Method

Subtract 9 or 11. Begin to add/subtract 19 or 21

35 – 9 = 26

+1

25   26   35
            -10

Use known number facts and place value to subtract
(partition second number only)

37 – 12 = 37 – 10 – 2

= 27 – 2

= 25

25   27   37

-1   - 1   -10

Pencil and paper procedures

```
   98
-  24
    4   (8-4)
   70   (90-20)
   74
```
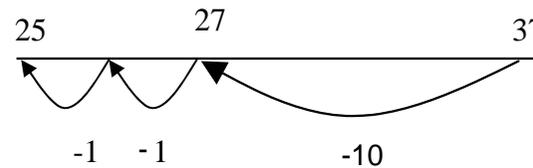
Use decomposition when appropriate

```
   9 2
-  3 8
   5 4   (explain what happens…see below)
```

90 + 2   →   80 + 12
30 + 8   -   30 + 8
                   50 + 4

```
    8  1
   9 2
-  3 8
   5 4
```

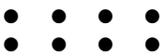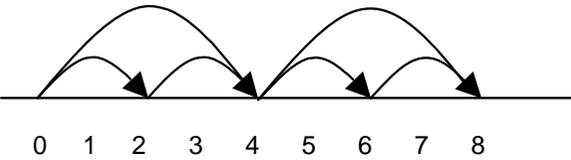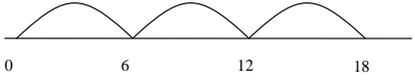| Subtraction		
Year 4 – Age Related Expectations	**Year 5 – Age Related Expectations**	**Year 6 – Age Related Expectations**
- subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate - estimate and use inverse operations to check answers to a calculation - subtraction two-step problems in contexts, deciding which operations and methods to use and why. $$\begin{array}{r} {}^{2}\cancel{3}{}^{4}5\,{}^{1}2 \\ -\,1\,7\,8 \\ \hline 1\,7\,4 \end{array}\qquad\begin{array}{r} {}^{4}\cancel{5}{}^{9}\cancel{0}{}^{9}\cancel{0}{}^{1}0 \\ -\ \ 4\,5\,7 \\ \hline 4\,5\,4\,3 \end{array}$$	- subtract whole numbers with more than 4 digits, including using formal written methods - subtract numbers mentally with increasingly large numbers - use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy - solve multi-step problems in contexts, deciding which operations and methods to use and why. **Pencil and paper procedures** Develop the use decomposition extend to up to 2 decimal places 48.42 – 37.61 = $$\begin{array}{r} 4\ \ {}^{7}\cancel{8}\ .\ {}^{1}4\ \ 2 \\ 3\ \ 7\ .\ \ 6\ \ 1 \\ \hline 1\ \ 0\ .\ \ 8\ \ 1 \end{array}$$ extend to up to 3 decimal places and numbers with different number of decimal places **302.63 – 178.124 =** $$\begin{array}{r} {}^{2}\cancel{3}{}^{9}0\,2\,.\,6\,{}^{2}\cancel{3}\,{}^{1}0 \\ -\ \ 1\,7\,8\,.\,1\,2\,4 \\ \hline 1\,2\,4\,.\,5\,0\,6 \end{array}$$	- perform mental calculations, including with mixed operations and large numbers - use their knowledge of the order of operations to carry out calculations involving the four operations - solve multi-step problems in contexts, deciding which operations and methods to use and why - use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. Algebra - use simple formulae - express missing number problems algebraically - find pairs of numbers that satisfy an equation with two unknowns

Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children are expected to be able to recall all multiplication facts to 12 × 12 by the end of year 4.

Multiplication		
Year 1 – Age Related Expectations	**Year 2 – Age Related Expectations**	**Year 3 – Age Related Expectations**
- solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. **Pictures and symbols** There are 3 sweets in one bag. How many sweets are there in 5 bags? *(Recording on a number line modelled by the teacher when solving problems)* Use of bead strings to model groups of.	- recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers - calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs - show that multiplication of two numbers can be done in any order (commutative) - solve problems involving multiplication using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts. **x = signs and missing numbers** 7 x 2 = □ □ = 2 x 7 7 x □ = 14 14 = □ x 7 □ x 2 = 14 14 = 2 x □ □ x ▽ = 14 14 = □ x ▽ **Arrays and repeated addition** ● ● ● ● 4 x 2 or 4 + 4 ● ● ● ● 2 x 4 or repeated addition 2 + 2 + 2 + 2 0 1 2 3 4 5 6 7 8	- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables - write and calculate mathematical statements for multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods - solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. **x = signs and missing numbers** Continue using a range of equations as in Level 2 but with appropriate numbers. Number lines 6 x 3 0 6 12 18 35 x 2 = 70 Partition

Doubling multiples of 5 up to 50

15 x 2 = 30

Partition

(10 x 2) + (5 x 2)

    20 +   10 = 30

x	30	5
2	60	10

Grid method
TU x U
23 x 7 is approximately 20 x 10 = 200

23 x 7 = 161

	T	U
	20	3
x		
7	140	21

Multiplication		
Year 4 – Age Related Expectations	**Year 5 – Age Related Expectations**	**Year 6 – Age Related Expectations**

Year 4 – Age Related Expectations

– recall multiplication tables up to 12 × 12

- use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers

- recognise and use factor pairs and commutativity in mental calculations

- multiply two-digit and three-digit numbers by a one-digit number using formal written layout

- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Pencil and paper procedures

Grid method
72 x 38 is approximately 70 x 40 = 2800

X	70	2	
30	2100	60	= 2160
8	560	16	= 576 +
			2736
			1

Estimate and check

Moving on to the long column method:

```
    72
  X 4
     8  (4 x 2)
  280  (4 x 70)
  288
```

Year 5 – Age Related Expectations

– identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers

- establish whether a number up to 100 is prime and recall prime numbers up to 19

- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

- multiply numbers mentally drawing upon known facts

- multiply and whole numbers and those involving decimals by 10, 100 and 1000

recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)

- solve problems involving multiplication including using their knowledge of factors and multiples, squares and cubes

- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

```
   3452
 X  31
   3452
 103560
 107012
```

Year 6 – Age Related Expectations

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

- perform mental calculations, including with mixed operations and large numbers

- identify common factors, common multiples and prime numbers

- use their knowledge of the order of operations to carry out calculations involving the four operations

- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Ratio & Proportion

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer - multiplication and division facts

- solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison

- solve problems involving similar shapes where the scale factor is known or can be found

- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

Algebra

- use simple formulae

- express missing number problems algebraically

- find pairs of numbers that satisfy an equation with two unknowns

```
   72
X  38
    16  (8 x 2)
   560  (8 x 70)
    60  (30 x 2)
  2100  (30 x 70)
  2736
```

Then the short column (formal) method:

```
        7   2
    X   3   8
        5   7   6
    2   1   6   0
    2   7₁  3   6
```

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Division		
Year 1 – Age Related Expectations	**Year 2 – Age Related Expectations**	**Year 3 – Age Related Expectations**
- solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. **Pictures / marks** 12 children get into teams of 4 to play a game. How many teams are there? 	- recall and use division facts for the 2, 5 and 10 multiplication tables - calculate mathematical statements for division within the multiplication tables and write them using division (÷) and equals (=) signs - show that division of one number by another must be done in the order stated - solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and division facts, including problems in contexts. **÷ = signs and missing numbers** $6 ÷ 2 = \square$ $\square = 6 ÷ 2$ $6 ÷ \square = 3$ $3 = 6 ÷ \square$ $\square ÷ 2 = 3$ $3 = \square ÷ 2$ $\square ÷ \nabla = 3$ $3 = \square ÷ \nabla$ **Understand division as sharing and grouping** **Sharing** – 6 sweets are shared between 2 people. How many do they have each? 	- recall and use division facts for the 3, 4 and 8 multiplication tables - write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods - solve problems, including missing number problems, involving division **÷ = signs and missing numbers** Continue using a range of equations as in Level 2 but with appropriate numbers. Consolidate yr 2 expectations: $18 ÷ 3$ can be modelled as: Sharing – 18 shared between 3 (see Level 2 diagram) Grouping - How many 3's make 18? **Remainders** $16 ÷ 3 = 5\ r1$ Sharing - 16 shared between 3, how many left over?
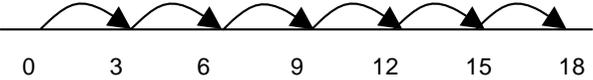
Grouping – How many 3's make 16, how many left over?
e.g.

0   3   6   9   12   15   16

Progressing to:

```
      28
  3 ⌐84
     30    3 × (10)
     54
     30    3 × (10)
     24
     24    3 × (8)
      0
```

6 ÷ 2 can be modelled as:

Grouping – There are 6 sweets. How many people can have 2 each? (How many 2's make 6?)

0   2   4   6

Division		
Year 4 – Age Related Expectations	**Year 5 – Age Related Expectations**	**Year 6 – Age Related Expectations**
- recall division facts for multiplication tables up to 12 × 12 - use place value, known and derived facts to divide mentally, including: dividing by 1; **Reinforce chunking becoming more efficient (e.g. × 10, x20)** 31 r1 3⟌94 30 3 x ⑩ 64 60 3 x ⑳ 4 3 3 x ① 1	- divide numbers mentally drawing upon known facts - divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context - divide whole numbers and those involving decimals by 10, 100 and 1000 - solve problems involving, squares and cubes - solve problems involving addition, subtraction, multiplication and division and a combination of these, - solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates. <u>Pencil and paper procedures</u> SHORT DIVISION METHOD 2 1 5 3 4⟌8 6 ²1 2 Write remainders or round if apprpriate	- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and - - interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context - divide numbers up to 4 digits by a two-digit number using the formal written method of short division where - - - appropriate, interpreting remainders according to the context - perform mental calculations, including with mixed operations and large numbers - use their knowledge of the order of operations to carry out calculations involving the four operations - solve problems involving addition, subtraction, multiplication and division - use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. Ratio & Proportion - solve problems involving the relative sizes of two quantities where missing values can be found by using integer - multiplication and division facts - solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison - solve problems involving similar shapes where the scale factor is known or can be found - solve problems involving unequal sharing and grouping using knowledge of fractions and multiples. Algebra

- use simple formulae
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns

Pencil and paper procedures

```
          291
45)13095
          90
          409
          405
            45
            45
```

Developing understanding of how to write remainders as fractions or decimals